Int. J. Heat Mass Transfer. Vol. 20, pp. 997–999. Pergamon Press 1977. Printed in Great Britain

THE NATURAL CIRCULATION SOLAR HEATER-MODELS WITH LINEAR AND NONLINEAR TEMPERATURE DISTRIBUTIONS

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(Received 30 January 1976 and in revised form 19 November 1976)

NOMENCLATURE

$\hat{A}$, cross section area;
c_p, specific heat of the fluid;
G, $\equiv \dfrac{\rho\beta g\,\hat{T}_{max}}{\hat{A}\,\hat{V}K}$, dimensionless parameter, equation (1), related to the Grashof number;
f, local flow resistance factor, defined by the head loss: $\hat{Q}f(\hat{s})\,d\hat{s}/\rho g[\hat{A}(\hat{s})]^2$ for an element of length $d\hat{s}$, $f = 8\pi\mu$ for a circular tube;
g, acceleration of gravity;
H, dimensionless parameter, related to the heat losses from the collector and the head losses of the entire system;
K, $= \dfrac{1}{\hat{L}_s}\oint \dfrac{f(\hat{s})\,d\hat{s}}{[\hat{A}(\hat{s})]^2}$, flow-resistance factor of the entire system;
L, dimensionless length;
$\hat{L}_s$, overall length of the circulation loop;
Q, dimensionless volumetric flow rate;
q, dimensionless heat flux per unit length;
q_0, dimensionless solar radiation heat flux per unit length, absorbed in the collector plate;
s, dimensionless coordinate along the circulation loop;
T, dimensionless temperature above the ambient;
T_H, dimensionless highest temperature in the system $[T_c(L_c)]$;
T_m, dimensionless mean temperature of the system;
$\hat{T}_{max}$, maximum possible temperature;
ΔT, dimensionless temperature difference along the collector and the tank;
$\hat{U}$, overall heat-transfer coefficient (per unit length);
U, $= \dfrac{\hat{U}\hat{L}_s}{\rho c_p \hat{A}\hat{V}}$, dimensionless overall heat-transfer coefficient;
$\hat{V}$, characteristic velocity;
Z, dimensionless height of the system;
z, dimensionless vertical coordinate.

Greek symbols

α, dimensionless parameter, equation (4), $1/\alpha$ representing dimensionless flow rate;
β, thermal expansion coefficient;
γ, dimensionless parameter, equation (4), representing energy losses or utilization of energy from the tank;
η, thermal efficiency;
λ, relative length of the tank;
μ, dynamic viscosity;
ξ, relative height of the system;
ρ, density of the fluid;
ϕ, tilt angle of the collector relative to the horizon.

Subscripts

c, collector;
t, tank.

Special notation

$\hat{}$, dimensional.

1. INTRODUCTION

THE NATURAL-CIRCULATION solar water heater, which consists of a flat plate collector, a storage tank and connecting pipes, is the most commonly used solar energy system. An exact theoretical evaluation of its performance requires a complex method of solving simultaneously the coupled momentum and energy differential equations [1]. In existing models for this system [2], temperature distributions in the collector and the tank are assumed linear and the mean temperature T_m is assumed equal in both. Accordingly, an overall energy balance yields T_m and the flow rate is then obtained by an overall momentum equation.

The present work outlines the method of solving the differential energy equation and the coupled momentum equation to obtain the steady state temperature distributions and the flow rate. This model can serve only as an approximation for the behavior of the system around noon-time, when variation of the impinging energy flux is rather weak and all the system components have already been heated-up. However, using this simplified model, it is still possible to evaluate the effects of the various system parameters and to determine the range of validity of the commonly used assumption of linear temperature distribution. Comparison with the latter model shows that it provides quite accurate results of flow rates, highest temperatures and efficiencies, for most of the practical range of system parameters. Rather large deviations are found at the limits of this range.

2. ANALYSIS

The system (see Fig. 1) is represented by a one-dimensional model, c.f. [1, 2], wherein the coordinate s is taken along the closed-circulation loop. $T(s)$ is the mean cross-sectional temperature and the flow rate Q is uniform. It is assumed that the heat-transfer coefficients and the properties of the fluid are constant, except for the buoyancy forces where the density depends on a constant expansion coefficient.

Dimensionless parameters and variables are chosen: lengths are scaled by the overall circulation length $\hat{L}_s$, temperatures by the maximal attainable temperature $\hat{T}_{max} = \hat{q}_0/\hat{U}_c$, and the flow rate by $\hat{A}\hat{V}$ where $\hat{A}$ is a typical cross-sectional area and $\hat{V} \equiv (g\hat{L}_s\beta\hat{T}_{max})^{1/2}$ is the characteristic velocity.

The momentum equation for the entire circulation loop is obtained by piecewise integration over the various parts. For laminar flow, the dimensionless form of the equation is:

$$Q = G\oint T\,dz. \tag{1}$$

The energy equation is written (non-dimensionally) for each part of the system:

$$Q\frac{dT}{ds} = \begin{cases} 0 & \text{(connecting pipes)} \\ U_c[1 - T_c(s)] \quad 0 \leqq s \leqq L_c & \text{(collector)} \\ -U_t T_t(s) \quad 0 \leqq s \leqq L_t & \text{(tank).} \end{cases} \tag{2}$$

Equations (2) are solved using Q as a constant (yet unknown) and the boundary conditions $T_c(L_c) = T_t(0)$, $T_c(0) = T_t(L_t)$ which follow from the uniformity and continuity of the

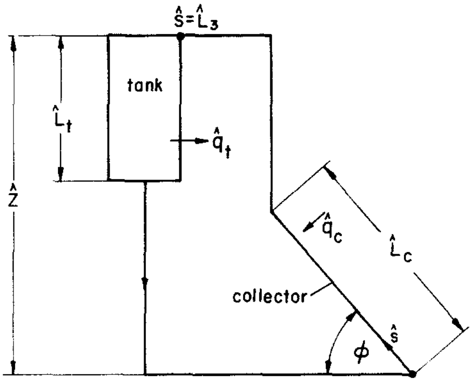

FIG. 1. Schematic arrangement of the natural circulation solar heater.

temperature profile, to yield:

$$T_c(s) = 1 - \frac{1-e^{-\gamma\alpha}}{1-e^{-\alpha(1+\gamma)}}\, e^{-\alpha s/L_c} \quad 0 \leq s \leq L_c \tag{3a}$$

$$T_t(s) = \frac{1-e^{-\alpha}}{1-e^{-\alpha(1+\gamma)}}\, e^{-\gamma\alpha s/L_t} \quad 0 \leq s \leq L_t \tag{3b}$$

where:

$$\alpha \equiv U_c L_c/Q, \quad \gamma \equiv U_t L_t/(U_c L_c). \tag{4}$$

Introduction of equations (3) into (1) and performing the integration, the following algebraic equation is obtained:

$$-\frac{H}{\alpha} + \sin\phi = \frac{1-e^{-\alpha}}{1-e^{-\alpha(1+\gamma)}}\left\{(1-e^{-\gamma\alpha})\left[\frac{1}{\alpha}\left(\sin\phi + \frac{\lambda}{\gamma}\right) - \xi\right] + \sin\phi - \lambda e^{-\gamma\alpha}\right\} \tag{5}$$

where $H \equiv U_c/G$, $\xi \equiv Z/L$, and $\lambda \equiv L_t/L_c$. The flow rate Q can be obtained by solving equation (5) for α [see equation (4)]. The solution depends on the five parameters γ, H, ξ, λ, and ϕ.

The thermal efficiency is defined by the rate of energy transfer from the collector to the tank:

$$\eta = \frac{\rho c_p \hat{Q} \Delta \hat{T}}{\hat{q}_0 \hat{L}_c} = \frac{\Delta T}{\alpha}. \tag{6}$$

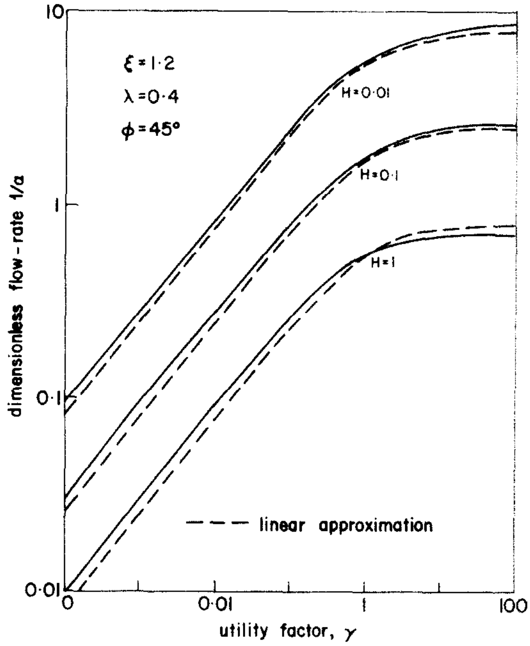

FIG. 2. The dimensionless flow rate $1/\alpha$ as a function of the utility factor γ, for various values of the parameter H.

The assumption of linear temperature profile (*the "linear model"*)

A simpler model for the system behavior is obtained by assuming linear temperature distributions in the collector and tank. Instead of equations (2), an overall energy balance is used:

$$\hat{L}_c \hat{q}_0 = \hat{U}_c \hat{L}_c \hat{T}_m + \hat{U}_t \hat{L}_t \hat{T}_m \tag{7}$$

which yields the non-dimensional mean temperature:

$$T_m = 1/(1+\gamma). \tag{8}$$

The temperature difference ΔT is obtained from an energy balance of the collector (or tank):

$$\Delta T = \alpha\gamma/(1+\gamma). \tag{9}$$

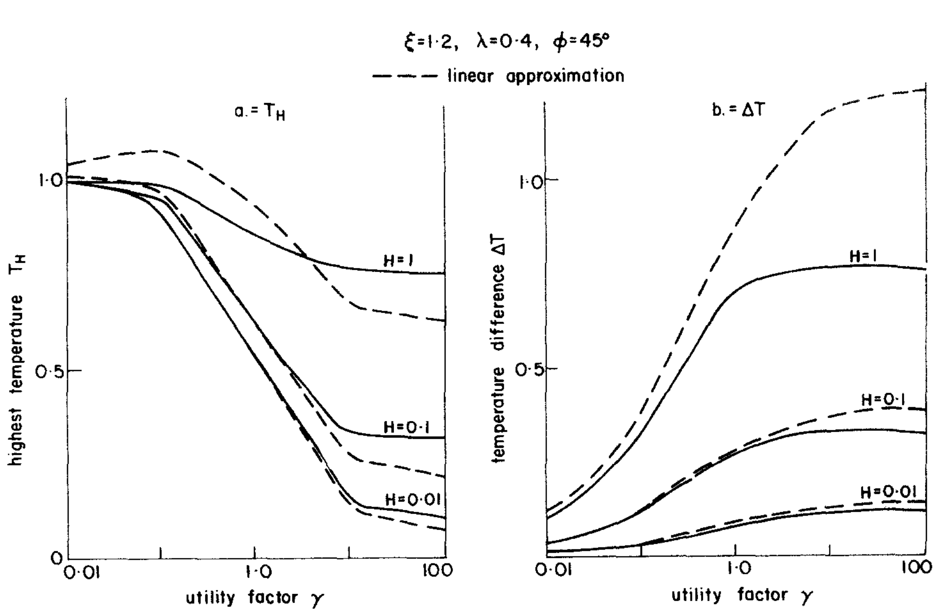

FIG. 3. The dimensionless highest temperature T_H (a) and temperature difference ΔT(b) as functions of the utility factor γ, for various values of the parameter H.

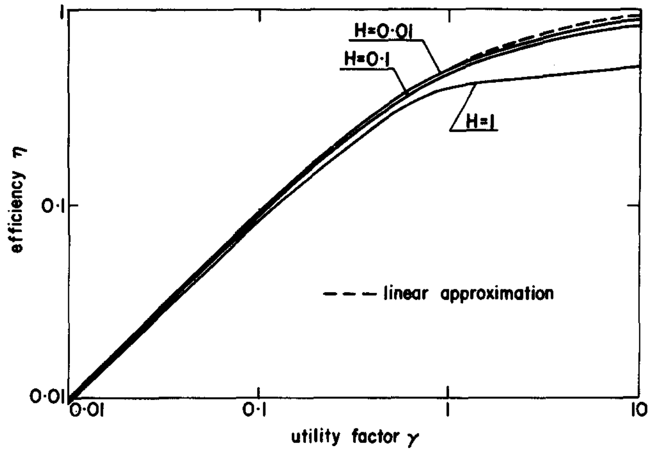

FIG. 4. The efficiency η as a function of the utility factor γ, for various values of the parameter H.

It is seen, then, that the efficiency [equation (6)] is given by:

$$\eta = \gamma/(1+\gamma). \tag{10}$$

Finally, the dimensionless flow rate $1/\alpha$ is obtained by introduction of the linear temperature profile and equation (9) into (1):

$$\frac{1}{\alpha} = \left\{\frac{\gamma[\xi - \frac{1}{2}(\lambda + \sin\phi)]}{(1+\gamma)H}\right\}^{1/2}. \tag{11}$$

3. RESULTS AND DISCUSSION

For a typical solar water heater the geometrical parameters are $\xi = 1.2$, $\lambda = 0.4$ and $\phi = 45°$. The parameter H, which includes both heat and head losses, is $0.01 \div 0.1$. The relative tank to collector heat loss parameter γ is typically 0.03 but would increase by orders of magnitudes if the system is operated in a primary-secondary cycle and the tank is actually a heat exchanger.

The dimensionless flow rate $1/\alpha$ is found by a numerical solution of equation (5) and by equation (11) of the linear model. The results shown in Fig. 2 indicate that the "linear model" can well serve as a first approximation for estimating the flow rate. Deviations between this model and the more exact one are 5–10% in the practical range of γ and H. For smaller and larger values of γ and for larger H the deviations increase. It is interesting to note that as H increases the curves of $1/\alpha$ vs γ for the two models intersect at the range $\gamma = 1$–10.

Highest temperature T_H obtained by equation (3a) or by (8) and (9) of the "linear model" are shown in Fig. 3(a). It is seen, again, that the "linear model" provides good estimates in the practical range of parameters. However, at the ends of this range the deviations become larger and even totally unrealistic: values of $T_H > 1$ (see Fig. 3a) mean temperatures which exceed the maximal possible ones. Moreover, the results for ΔT illustrated in Fig. 3(b) yield $\Delta T > 1$ for the linear model, which is impossible, again. As can be seen from Figs. 3(a), (b) the "linear model" predicts temperatures below the ambient for large values of γ and H.

The results for the efficiency are shown in Fig. 4. The same behavior of the "linear model" is observed: it deviates significantly from the more exact one for large values of γ and H only.

As an example, consider the performance of the typical solar water heater with $H = 0.01$, for which 50% efficiency corresponds to the utility factor $\gamma = 1$ (see Fig. 4). The dimensionless flow rate for this case is $1/\alpha = 5.7$ (Fig. 2), and equation (4) leads then to $\hat{Q} = 4.4 \times 10^{-5}\ \mathrm{m^3/s}$. The highest temperature is $T_H = 0.54$ (Fig. 3), meaning 38°C above the ambient. If the same system is operated without utilization of energy, $\gamma = 0.03$ represents heat losses from the tank. The flow rate for this case is found to be $1.1 \times 10^{-5}\ \mathrm{m^3/s}$ and the highest temperature $T_H = 69°\mathrm{C}$.

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